$$A = T_2 - T_1$$

Predavanje 26. oktober 2020

Izrek o delu in energiji

$$\vec F = -\operatorname{grad} U \; ; \quad U = U(P)$$

Potencialna sila, konzervativna sila.

Izrek 1. Izrek o energiji. *Naj bo $\vec F$ konzervativna sila. Potem je vsota kinetične in potencialne energije konstanta gibanja.*

$$A = \int_{P_1}^{P_2} \vec F \cdot d\vec r = -\int_{P_1}^{P_2} \operatorname{grad} U \cdot d\vec r = -\int_{t_1}^{t_2} \operatorname{grad} U(\vec r(t)) \cdot \vec v \, dt = -\int_{t_1}^{t_2} \frac{dU}{dt}\, dt$$

$$U = U(P) = U(\vec r) \qquad P_1 = \vec r(t = t_1) \qquad d\vec r = \vec v \cdot dt$$

$$\frac{d\, U(\vec r(t))}{dt} = \frac{\partial U}{\partial x}\dot x + \frac{\partial U}{\partial y}\dot y + \frac{\partial U}{\partial z}\dot z = \operatorname{grad} U \cdot \vec v$$

$$T_2 - T_1 = A = -U_2 + U_1 \qquad T_2 + U_2 = T_1 + U_1 = E_0$$

$$\vec F = f(x) \; ; \quad U(x)$$

$$F_1, F_2 \quad | \quad U$$

$$F_1 = -\frac{\partial U}{\partial x}, \quad F_2 = -\frac{\partial U}{\partial y}$$

$$\operatorname{rot} \vec F = \vec 0$$

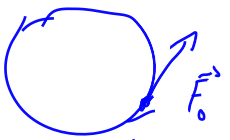
$$\Updownarrow$$

potencialna sila

$$U = U(\vec r, t) \; ; \quad \frac{dU}{dt} = \operatorname{grad} U \cdot \vec v + \frac{\partial U}{\partial t}$$

Osnovni primeri sil

- Gibanje v polju konstantne sile $\vec F = \vec F_0 = m\vec g$.



$$\vec F_0 = f_0 \vec e_\varphi \qquad \vec v = r\dot\varphi \vec e_\varphi$$

$$A = \int_{P_1}^{P_2} \vec F_0 \cdot d\vec r = r f_0 \int_{P_1}^{P_2} d\varphi = r f_0 (\varphi_2 - \varphi_1)$$

$$\vec F = -\operatorname{grad} U = -\frac{\partial U}{\partial r}\vec e_r - \frac{1}{r}\frac{\partial U}{\partial \varphi}\vec e_\varphi$$

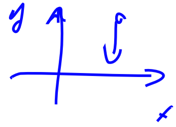
$$m\vec a = \vec F$$

$$m r \ddot\varphi = f_0 = -\frac{dU}{d\varphi}\frac{1}{r}$$

$$m r^2 \ddot\varphi = r f_0 = -\frac{dU}{d\varphi} \; / \dot\varphi \implies -\frac{1}{2} m r^2 (\dot\varphi^2)^{\cdot} = \frac{dU}{d\varphi}\dot\varphi = \frac{dU}{dt}(\varphi)$$

$$U = -\frac{1}{2} m r^2 \dot\varphi^2 + U_0 = -\frac{1}{2} m v^2 + U_0$$

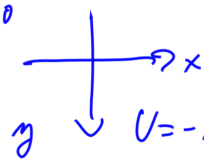
$U = -m(g_1 x + g_2 y + g_3 z)$; $-\frac{\partial U}{\partial x} = m g_1$



$U = mgy$

$\vec g = g_1 \vec i + g_2 \vec j + g_3 \vec k$

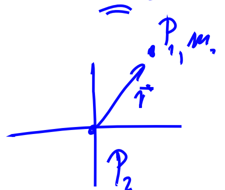
$U = -m\vec g \cdot \vec r + \cancel{U_0}$

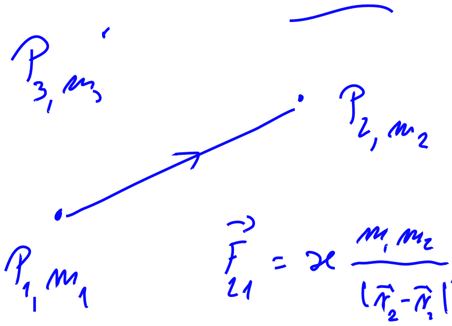


$U = -mgy$

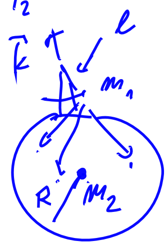
- Gravitacijska sila $\vec F = -\frac{\kappa m_1 m_2}{r^2}\vec r/r$, potencial je

$$U = -\kappa m_1 m_2/r.$$





$\vec F_{21} = \varkappa \frac{m_1 m_2}{|\vec r_2 - \vec r_1|^2} \frac{\vec r_2 - \vec r_1}{|\vec r_2 - \vec r_1|}$ +



$\frac{\vec r}{|\vec r|} = \vec k$ $\quad \vec F = -m_1 g \vec k \quad g = \frac{\varkappa m_2}{R^2}$

Določi potencial za $-\frac{\vec r}{|\vec r|}$

Iščemo U: $\operatorname{grad} U = -\frac{\vec r}{|\vec r|}\frac{1}{r^2}$

$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = \frac{\partial}{\partial x}\frac{1}{\sqrt{x^2+y^2+z^2}} = -\frac{1}{2}\frac{2x}{(x^2+y^2+z^2)^{3/2}}$

$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{x}{|\vec r|^3} = -\frac{x}{r^3}$

$\frac{\partial}{\partial y}\left(\frac{1}{r}\right) = -\frac{y}{r^3}$

$\boxed{\operatorname{grad} U = -\frac{1}{r^3}\vec r \Leftarrow U = \frac{1}{r}}$

- Električna sila.

e_2

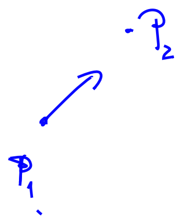
e_1

$e_1 e_2 > 0$

sila je odbojna

$e_1 \cdot e_2 < 0$ privlačna

$\vec F_{21} = -\varepsilon \frac{e_1 e_2}{|\vec r_1 - \vec r_2|^2} \frac{\vec r_2 - \vec r_1}{|\vec r_2 - \vec r_1|}$

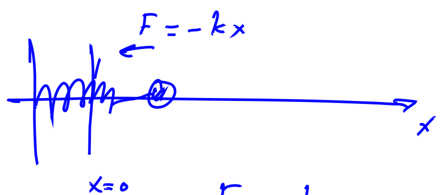


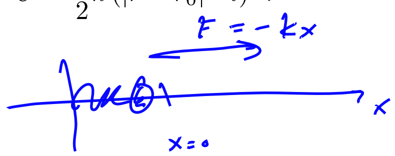
- Sila vzmeti, Hookova sila.

$$\vec F = -k\frac{\vec r - \vec r_0}{|\vec r - \vec r_0|}\left(|\vec r - \vec r_0| - l\right)$$

Potencial je

$$U = \frac{1}{2}k\left(|\vec r - \vec r_0| - l\right)^2.$$





$F = -kx \qquad U = \frac{1}{2}kx^2 \qquad x$ raztezek

Splošno l

$F = -k(x-l)$; $U = \frac{1}{2}k(x-l)^2 \qquad x$ dolžina vzmeti

$\vec F = -k\frac{\vec r - \vec r_0}{|\vec r - \vec r_0|}\left(|\vec r - \vec r_0| - l\right)$

$$\vec F_u = -\frac{\vec v}{|\vec v|}\, f(|\vec v|) \qquad f>0$$

linearni zakon

$$f(|\vec v|) = \alpha |\vec v| \qquad \alpha > 0$$

$$\vec F_u = -\alpha \vec v$$

- Sila upora, sila trenja.

Kvadratni

$$f(|\vec v|) = \beta |\vec v|^2$$

$$\vec F_u = -\beta |\vec v| \vec v = -\beta \sqrt{\vec v \cdot \vec v}\, \vec v$$

$$[F_u] = N$$

$$[F_u] = [f] = [\beta] \cdot \frac{m^2}{s^2}$$

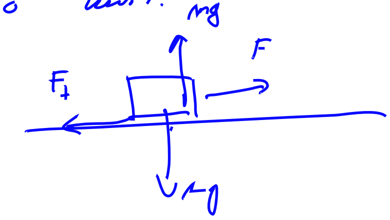
$$[\beta] = \frac{N s^2}{m^2} = \frac{kg\, m}{m^2} = \frac{kg}{m}$$

Sila trenja

$$\vec F_t = -\gamma \frac{\vec v}{|\vec v|} \qquad \gamma > 0 \text{ konst.}$$

$$\gamma = mgk$$

koeficient trenja



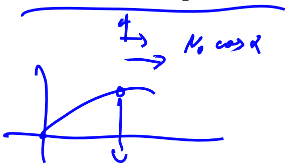
PREMOČRTNO GIBANJE

k brezdimenzijski

$k=1$ veliko trenje

$k > \frac{1}{2}$

Tir leži na premici.



Definicija 1. *Gibanje je premočrtno, če ima vektor pospeška konstantno smer.*

$$\vec a = \vec a_0 \qquad \dot{\vec v} = \vec a_0 \Rightarrow \vec v = \vec a_0 t + \vec v_0$$

Kvadraturna rešitev

$m\vec{a} = \vec{F}$ $\qquad$ $m\ddot{x} = f(t, x, \dot{x})$

$\vec{F} = f\vec{\imath}$ $\qquad$ $\underline{x(t=t_0) = x_0}$, $\underline{\dot{x}(t=t_0) = v_0}$

$\boxed{m\ddot{x} = f(x)}$ $\qquad$ Ali obstaja U tako, da je $U(x)$

$f = -\frac{dU}{dx} \Longleftrightarrow U = -\int_{x_0} f(\xi)\,d\xi + U_0$

Ali je res $-\frac{dU}{dx} = f(x)$ $\qquad$ f zvezna

Pišimo $\vec{F} = f\vec{\imath}$ in naj bo $f = f(x)$ zvezna funkcija. Potem je $T + V = E$ je integral gibanja.

f zvezna $\Rightarrow$ f je potencialna $\qquad$ $U = -\int_{x_1}^{x} f(\xi)\,d\xi + U_0$

$T + U = E_0$

$\boxed{\frac{1}{2}m\dot{x}^2 + U(x) = E_0}$

$\dot{x} > 0$

$\dot{x} < 0$

$\dot{x}^2 = \frac{2}{m}(E_0 - U(x))$ $\qquad$ $\frac{dx}{dt} = \dot{x} = \pm\sqrt{\frac{2}{m}(E_0 - U(x))}$

$\pm$: $\operatorname{sgn}\dot{x}$

$\underline{\underline{\operatorname{sgn}\dot{x}\int_{x_1}^{x} \frac{dx}{\sqrt{\frac{2}{m}(E_0 - U(x))}}}} = \int_{t_1}^{t} dt = t - t_0$

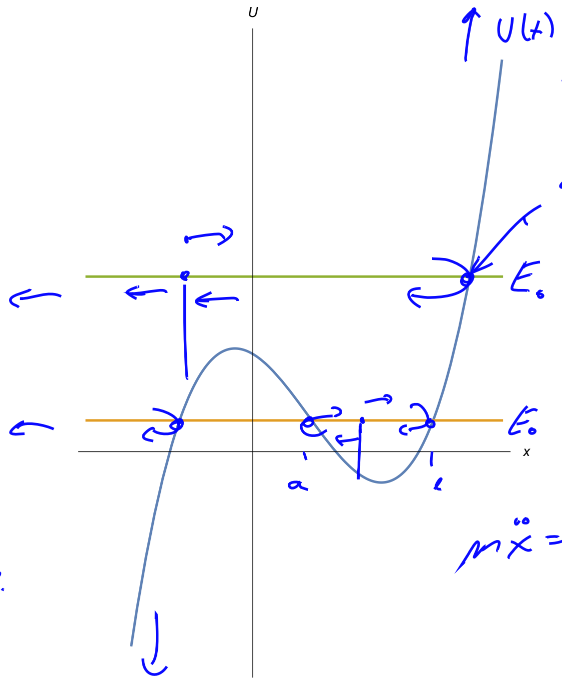
$\dot{x} \neq 0$

$t = t(x) \Longleftrightarrow \underline{x = x(t)}$

$$t = t_0 + \operatorname{sgn}\dot{x}\int_{x_0}^{x} \frac{dx}{\sqrt{\frac{2}{m}(E - V(x))}}.$$

Kvalitativna analiza gibanja

$\frac{1}{2} m \dot{x}^2 + U(x) = E_0$



točk. obrata

$\dot{x} = 0$

$\ddot{x} \neq 0$

$\dot{x} < 0$

$m\ddot{x} = F = -\frac{dU}{dx}$

Na $[a, b]$ je gibanje periodično.

Slika 1: Graf potenciala.

$$T = \sqrt{2m}\int_a^b \frac{dx}{\sqrt{E-U(x)}}.$$

Perioda periodičnega gibanja

Lema 1. *Velja*

(a)

$$\int_a^b \frac{dx}{\sqrt{(b-x)(x-a)}} = \pi$$

Harmonični oscilator

Harmonična aproksimacija

$$T \approx 2\pi\sqrt{m/U''(x_0)}.$$

Duffingov oscilator

$$U = \frac{1}{2}\alpha x^2 + \frac{1}{4}\beta x^4, \quad \alpha\beta < 0.$$