Direct limit

Naloga 6.1 Let $G_n = \mathbb{Z}$ for all $n \in \mathbb{N}$ and let $\varphi_{n,n+1}, \psi_{n,n+1}, \rho_{n,n+1} \colon G_n \to G_{n+1}$ be defined by $\varphi_{n,n+1}(x) = 0$, $\psi_{n,n+1}(x) = 2x$ and $\rho_{n,n+1}(x) = (n+1)x$. Find $\varinjlim(G_n, \varphi_{n,n+1})$, $\varinjlim(G_n, \psi_{n,n+1})$ in $\varinjlim(G_n, \rho_{n,n+1})$.

Naloga 6.2 Let $x_0 \in X_0 \subset X_1 \subset ... X$ be a sequence of subspaces and let $X = \bigcup_{n=1}^{\infty} X_n$. For n < m let $i^{n,m} \colon X_n \hookrightarrow X_m$ and $i^n \colon X_n \hookrightarrow X$. If for every compact set $K \subset X$ exists $n \in \mathbb{N}$ such that $K \subset X_n$, then $\pi_1(X)$ together with homeomorphisms $i^n_\# \colon \pi_1(X_n) \to \pi_1(X)$ is the direct limit.

Naloga 6.3 Let $T_0 \subset \mathbb{R}^3$ be a torus and $f: \mathbb{R}^3 \to \mathbb{R}^3$ be homeomorphism such that $f(T_0) \subset \operatorname{Int} T_0$ is as it is shown on the picture.



Let $T_{n+1} = f(T_n)$ and let $T = \bigcap_{n=1}^{\infty} T_n$. The space T is called **diadic solenoid**. Find $\pi_1(\mathbb{R}^3 - T)$.

Naloga 6.4 Let $T_0 \subset \mathbb{R}^3$ be a torus and let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be a homeomorphism such that $f(T_0) \subset \operatorname{Int} T_0$ is Whitehead link as it is shown on the picture.



Let $T_{n+1} = f(T_n)$. The intersection $T = \bigcap_{n=1}^{\infty} T_n$ is called **Whitehead continua**. Find $\pi_1(\mathbb{R}^3 - T)$.