

Krovi

Naloga 7.1 Let $\mathbb{S}_a(r) \subset \mathbb{R}^2$ be the circle $\{(x, y) \mid d((x, y), a) = r\}$. Let

$$X = \mathbb{S}_{(-2,0)}(1) \cup \mathbb{S}_{(0,0)}(1),$$

$$Y = \mathbb{S}_{(-2,0)}(1) \cup \mathbb{S}_{(0,0)}(1) \cup \mathbb{S}_{(2,0)}(1).$$

Find out if $p_i: Y \rightarrow X$ defined by

$$p_1(x, y) = \begin{cases} (-|x|, y), & |x| \geq 1, \\ (\cos(\pi x), \sin(\pi x)), & |x| \leq 1, \end{cases}$$

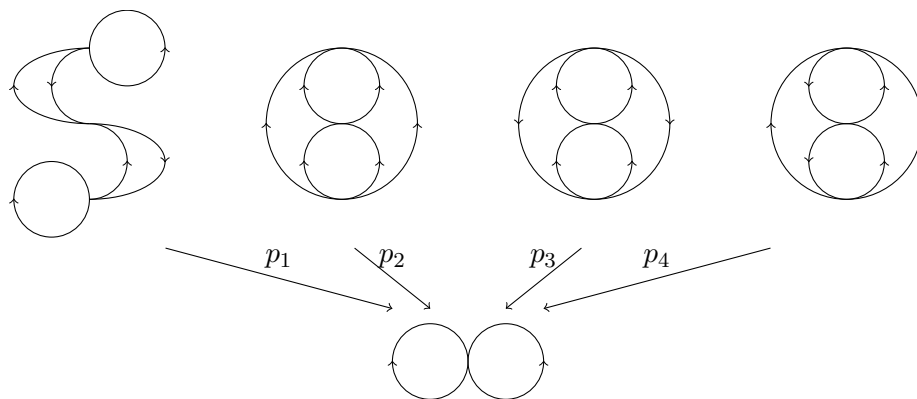
$$p_2(x, y) = \begin{cases} (-|x|, y), & |x| \geq 1, \\ (\cos(\pi x), \sin(\pi x)), & |x| \leq 1, y \geq 0, \\ (\cos(\pi x), -\sin(\pi x)), & |x| \leq 1, y \leq 0, \end{cases}$$

$$p_3(x, y) = \begin{cases} (-|x|, y), & |x| \geq 1, \\ (\cos(\pi x), -\sin(\pi x)), & |x| \leq 1, y \geq 0, \\ (\cos(\pi x), \sin(\pi x)), & |x| \leq 1, y \leq 0, \end{cases}$$

$$p_4(x, y) = \begin{cases} (|x| - 2, y), & |x| \geq 1, \\ (-\cos(\pi x) - 2, \sin(\pi x)), & |x| \leq 1, y \geq 0, \\ (-\cos(\pi x) - 2, -\sin(\pi x)), & |x| \leq 1, y \leq 0. \end{cases}$$

is covering projection? Which covers are isomorphisms?

Naloga 7.2 Let $X = \mathbb{S}_{(-1,0)}(1) \cup \mathbb{S}_{(1,0)}(1)$ and $p_i: Y_i \rightarrow X$ as in the figure.



Find out which p_i is covering projection? Which covers are isomorphisms?

Naloga 7.3 Let $f, g: \mathbb{C} - \{0\} \rightarrow \mathbb{S}^1 \times \mathbb{S}^1 \subset \mathbb{C}^2$ be defined by $f(z) = (\frac{z}{|z|}, e^{i|z|})$ in $g(z) = (\frac{z}{|z|}, e^{i \ln |z|})$. Are f and g covering projections?

Naloga 7.4 Let $G = \mathbb{Z}$ acts on $X = \mathbb{R}^2 - \{(0,0)\}$ by $n(x, y) = (2^n x, 2^{-n} y)$. Show that $q: X \rightarrow X/G$ is covering projection. Show that X/G is not Hausdorff space.

Naloga 7.5 Let $p: \tilde{X} \rightarrow X$ be a covering projection and let X be Hausdorff space. Prove that $\tilde{X}$ is Hausdorff space.

Naloga 7.6 Let $f: \tilde{X} \rightarrow X$ be a covering projection.

1. If $\tilde{X}$ is second countable then X is second countable.
2. If $\tilde{X}$ is connected and X is second countable then $\tilde{X}$ is second countable.

Naloga 7.7 Let X , Y , and Z be locally path connected spaces and let $p: X \rightarrow Y$, $q: Y \rightarrow Z$, and $r = q \circ p: X \rightarrow Z$.

1. If p and r are covering projections then q is covering projection.
2. If q and r are covering projections and Y is connected then p is covering projection.
3. If p and q are covering projections and q is a finite-sheeted covering then r is covering projection.
(The assumption on q is necessary as it is shown in the exercise 7.8.)

Naloga 7.8 For all $m, n \in \mathbb{Z}$ define $K_n^m = \{z \in \mathbb{C} \mid |z - (\frac{n}{|n|+1} + 3mi)| = \frac{1}{|n|+1}\}$. Let

$$\begin{aligned} Z &= \cup_{n \geq 0} K_n^0 \quad (\text{Hawaiian earring}), \\ Y &= (\cup_{m \in \mathbb{Z}} \cup_{n \in \mathbb{N}} K_n^m) \cup (\{1\} \times i\mathbb{R}), \\ X &= (\cup_{m \in \mathbb{Z}} \cup_{\substack{n \in \mathbb{Z} \\ |n| \neq 1}} K_n^m) \cup (\{-1, 1\} \times i\mathbb{R}). \end{aligned}$$

Let $\psi: \mathbb{C} \rightarrow \mathbb{C}$ be define by $\psi(x + iy) = |x| + iy$. Maps $q: Y \rightarrow Z$ and $p: X \rightarrow Y$ are defined by

$$q(z) = \begin{cases} z - 3mi & , z \in K_n^m, \\ e^{\frac{2}{3}\pi(z-1)} & , z \in \{1\} \times i\mathbb{R}, \end{cases}$$

and

$$p(z) = \begin{cases} \psi(z), & z \in \{-1, 1\} \times i\mathbb{R}, \\ \psi(z), & z \in K_n^m, |n| > |m+2| \\ \frac{|n|+1}{|n|}(\psi(z) - \frac{|n|}{|n|+1}) + \frac{|n|-1}{|n|}, & z \in K_n^m, 2 \leq |n| \leq |m+2|, \\ \frac{1}{|m|+1}((z - 3mi)^2 + 1) + 3mi, & z \in K_0^m. \end{cases}$$

Prove that p and q are covering projections but the compositum $q \circ p$ is not.